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Title of the Book: **Non-Linear Time Series**

The major subject of the book is Time series. It is intended for postgraduate students who have good knowledge of linear time series models and would like extend their knowledge to non-linear time series. A good knowledge of probability and statistics as well as some knowledge of Hilbert spaces is required. Some of the sections and chapters may require good knowledge of advanced material such as principles of Markov chain Monte Carlo methods .

There are quite a few books on Nonlinear time series giving emphasis on different aspects of the subject. Some examples are:

Tong(1990) Non-linear Time series Oxford science publications

Fan and Yao(2003) Nonlinear time series, Springer Verlag

Prado and West(2010) Time series Modeling Computation and inference Chapman and Hall

The proposed manuscript differs from above books in two aspects: First, there is relation between nonlinearity and heavy tails. the book aims to explain this relationship. Second, there has been advances in integer time series models spread over many journals. we aim to bring together all the material on the subject. We tried to balance the probabilistic and inferential aspects of nonlinear processes, but due to general difficulties on inference for nonlinear models, probabilistic aspects have a bigger share in the book.

## Contents

<b>1</b>	<b>Introduction</b>	<b>11</b>
1.1	Why we need nonlinear models . . . . .	11
1.1.1	Some examples of time series . . . . .	14
1.1.2	Some examples of integer time series . . . . .	22
1.1.3	Processes that show limit cycle . . . . .	25
1.1.4	Bilinear processes with Gaussian and heavier tailed innovations	26
<b>2</b>	<b>Probability structure of time series</b>	<b>35</b>
2.1	Introduction . . . . .	35
2.2	Probability structure of time series . . . . .	36
2.3	Moments of time series . . . . .	41
2.4	Gaussian Processes . . . . .	48
2.4.1	Some properties of Gaussian processes . . . . .	48
2.5	Stationary Processes . . . . .	50
2.6	The space of random variables with finite second moments: $L^2(\Omega, \mathcal{F}, P)$ and optimal predictions . . . . .	56
2.6.1	Hilbert spaces and the projection theorem . . . . .	56
2.6.2	Stationary time series with finite second order moments . . . . .	60
2.6.3	Minimum mean squared error prediction of a stationary process	63
2.7	Linear representations and linear models . . . . .	67
2.8	Spectral representations . . . . .	69
2.8.1	Spectral representations for deterministic functions; Fourier analysis . . . . .	69
2.8.2	Spectral representations for stationary time series . . . . .	73
2.8.3	Geometric interpretation of Fourier representations . . . . .	82
<b>3</b>	<b>Probabilistic Aspects of NL Processes</b>	<b>87</b>
3.1	Linear representations . . . . .	87
3.2	Non-linear representations . . . . .	105

3.3	Sensitive dependence on initial conditions, Lyapunov exponents . . .	114
3.4	Limit cycles . . . . .	121
3.5	Time reversibility . . . . .	122
3.6	Invertibility . . . . .	123
3.7	Tail behavior . . . . .	124
3.7.1	Extreme Value Theory . . . . .	124
3.7.2	Tail behavior of linear processes . . . . .	126
3.8	Connection between non-linearity and heavy tails . . . . .	131
3.8.1	Extremal properties of certain types of nonlinear difference equations . . . . .	133
3.8.2	Tails of Bilinear processes . . . . .	139
3.8.3	Tails of GARCH processes . . . . .	142
3.8.4	The relation between the extremes of finite order volterra series and bilinear proceses . . . . .	144
3.9	Linear models at the presence of heavy tails . . . . .	149
4	Models for Nonlinear Time Series . . . . .	153
4.1	Nonlinear models . . . . .	153
4.1.1	Parametric models for the conditional mean . . . . .	154
4.1.2	Exponential autoregressive models: . . . . .	164
4.1.3	Parametric models for the conditional variance . . . . .	165
4.1.4	Mixed models . . . . .	166
4.1.5	Generalized State space models . . . . .	173
4.1.6	Max-stable moving average processes . . . . .	183
4.2	Some statistical aspects . . . . .	185
4.2.1	Identification of nonlinearity . . . . .	185
4.2.2	Checking for Invertibility and stationarity . . . . .	193
4.3	Methods of Inference . . . . .	201
4.3.1	Least square and likelihood methods . . . . .	201
4.3.2	Estimating functions . . . . .	205
4.4	Bayesian methods . . . . .	223
5	Univariate GARCH-type models . . . . .	239
5.1	Introduction . . . . .	239
5.2	Basic properties of univariate GARCH-type models . . . . .	248
5.2.1	Stationarity and ergodicity . . . . .	248
5.2.2	Moment properties . . . . .	255
5.2.3	Auto-covariance/auto-correlation structure . . . . .	261
5.2.4	Long memory . . . . .	266

5.3	Parameters estimation . . . . .	270
5.3.1	Quasi-maximum likelihood . . . . .	271
5.3.2	Whittle estimation . . . . .	276
5.3.3	Bayesian approach . . . . .	280
5.4	Extremal behavior . . . . .	286
5.4.1	Application . . . . .	290
6	Models for integer-valued time series	301
6.1	Introduction . . . . .	301
6.2	Integer-valued ARMA models . . . . .	306
6.2.1	INAR(1) model . . . . .	307
6.2.2	INAR(p) model . . . . .	312
6.2.3	INMA(q) model . . . . .	315
6.3	Model selection . . . . .	317
6.4	Parameters estimation . . . . .	322
6.4.1	Moment-based estimators . . . . .	322
6.4.2	Conditional Least Squares . . . . .	324
6.4.3	Conditional Maximum Likelihood Estimation . . . . .	327
6.4.4	Bayesian approach . . . . .	329
6.5	Extremal behavior . . . . .	332
6.5.1	Some extensions . . . . .	337

**Marketing:**

**Disciplines addressed in the book:** Time series (primary) , stochastic processes (secondary).

The manuscript is intended as lecture notes volume.

There will be some examples, but no exercise sets are planned.

There should be no software issue, since the examples will be carried using the R software.

**Comparisons:**

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**Reviewers:**

Peter Diggle and T Subba Rao

**Schedule:** We have reasonably polished chapters ready to review and the pdf file is attached. Some further work is still needed, such as addition of some examples, polishing the language, style and overall debugging. All these can be done in a relatively short period of time. A conservative estimate would be 3-4 months.

**References:** Full references are included

**Production:** The manuscript has approximately 360 pages, including the references and graphs. There will be no color printing.