The importance of the generalized Pareto distribution in the extreme value theory

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Outline of the talk

1. A brief account of my scientific interests;
2. Elementary concepts about the Extreme Value Theory (EVT);
3. Methods for estimating the parameters of the generalized Pareto distribution (GPD);
4. Application of the extreme value theory to some financial data.
5. Comments and some conclusions.

A brief account of my scientific interests

- EVT - application point of view.
  - Forest fires - the area burned by large wildfires that occurred in Portugal between 1984 and 2004;
  - Medical sciences - the large total cholesterol levels observed in the 20 districts of Portugal;
- EVT - theoretical approach - review of methods to estimate the parameters of the generalized Pareto distribution (GPD). It was a joint work with Samuel Kotz (2010 a) and b)).
- Bayesian hierarchical models for modeling wildfires.
- Nonlinear time series models.

Very extreme events (large or small) frequently happen in various areas such as

- Economy - sudden changes in some economical variables due, for instance, to political measures;
- Finance - abrupt "jumps"observed in the stock market;
- Wildfires;
- Meteorology - rainfall (floods / droughts), extreme wind events (e.g., hurricanes, tornados); temperatures (high / low).
- Geology / Seismic activity - earthquakes;
- Hydrology - river level (high / low);
- Oceanography - sea level.

Floods

Figure 1: Flood occurred in Madeira, Portugal, February 2010

Source: Google images

Severe droughts

Figure 2: Severe drought in Canada (right)

Source: Google images

Hurricanes

Figure 3: Sky of Mississipi (USA) during the Hurricane Katrina, August 2005

Source: Google images

Earthquakes

Figure 4: Niigata-Ken Chuetsu, Japan, 2004

Source: Google images

Portuguese Stock market Index (PSI 20)

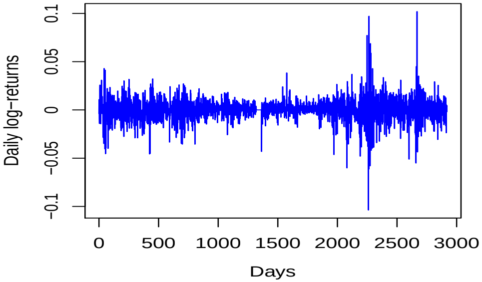

Figure 5: Daily log-returns at closing time, from 24 January 2000 to 29 April 2011

Questions that can be raised:

- What is the probability that the maximum value observed will be surpassed in the future?
- Are the large negative log-returns similar to the large positive log-returns?

Let X_1, X_2, ..., X_n be n independent and identically distributed (iid) random variables (rvs) with unknown cumulative distribution function (cdf) F. The objective might be to make inferences about the probability that the rv X will exceed a large value x. Then, attention focuses on estimating tail probabilities

$$P(X > x) = 1 - F(x).$$

We may also be interested on estimating extreme quantiles. In this situation the issue lies on determining a real number x_q such that:

$$P(X > x_q) = 1 - F(x_q) = q,$$

for a very small probability q. Commonly values of q much smaller than $1/n$ are the most interesting to consider.

Let $\{X_1, X_2, ...\}$ be again iid rvs with unknown cdf F. The cdf of $M_n = \max(X_1, X_2, ..., X_n)$ is given by

$$F_{M_n}(x) = P(M_n \leq x) = P(X_1 \leq x, X_2 \leq x, ..., X_n \leq x)$$

$$= \prod_{i=1}^{n} P(X_i \leq x) = [F(x)]^n.$$

What are the problems?

1. if F is unknown, how can we possibly figure out the distribution of the maximum of the n rvs?
2. $$\lim_{n\to\infty} F_{M_n}(x) = \lim_{n\to\infty} [F(x)]^n = \begin{cases} 0 & , F(x) < 1 \\ 1 & , F(x) = 1 \end{cases}$$

How can we overcome these issues?

If $\{a_n > 0\}$ and $\{b_n\}$ are sequences of constants (attraction coefficients) such that:

$$\lim_{n\to\infty} P\left(\frac{M_n - b_n}{a_n} \leq x\right) = \lim_{n\to\infty} F^n(a_n x + b_n) = G(x),$$

then $G(.)$, which is a non-degenerate cdf, belongs to one of the following families:

- Type I - Gumbel
- Type II - Fréchet
- Type III - Weibull

These three distribution can be unified in terms of the Generalized Extreme Value distribution (GEV) with cdf given by:

$$G\left(x \mid \xi, \mu, \sigma\right) = \exp\left[-\left(1-\xi\frac{x-\mu}{\sigma}\right)^{1/\xi}\right], 1-\xi\frac{x-\mu}{\sigma} > 0,$$

where ξ, μ ($\xi, \mu \in \mathbb{R}$) and σ ($\sigma > 0$) are respectively the shape, the location and the scale parameters. In the previous theorem, if G is given as above, then the df F is said to belong to the domain of attraction of the GEV distribution.

Distribution	Domain of attraction	ξ	Type of distributions
Uniform and Beta	Weibull	$\xi > 0$	short-tailed finite upper bound $\mu + \sigma/\xi$
Exponential, Gamma, Normal and Lognormal	Gumbel	$\xi = 0$	light to reasonably heavy
Pareto, Loggamma, t-Student and Burr	Fréchet	$\xi < 0$	heavy-tailed

Table 1: Some common distributions and their domains of attraction

Heavy tailed occur quite frequently when modeling *e.g.*

- insurance claims;
- stock market gains and losses.

Heavy tailed distribution or subexponential distributions, using insurance terminology, can be expressed as

$$\lim_{x\to\infty} \frac{P\left(\sum_{i=1}^{n} X_i > x\right)}{P\left(\max(X_1, X_2, ..., X_n) > x\right)} = 1, \forall n \geq 2$$

Meaning:

The tails of the distribution of the sum and of the maximum of the first n claims have, asymptotically, the same order (see Embrechts et al. (2003)).

The annual maxima method consists on partitioning an iid sample in blocks of equal size and retrieving the maximum in each block. In general the blocks consist on the observations recorded during the period of one year, although other options are possible. The sample of maxima is supposed to follow the GEV distribution, as the sample size increases.

Drawbacks

1. sometimes the temporal structure of the underlying phenomenon is not obvious;
2. waste of information;
3. slow convergence rate to the GEV observed in some distributions.

There is also the method of the largest observations.

The Peaks over threshold (POT) consists on fitting a model to the excesses (or exceedances) above a sufficiently high threshold u. This sample of excesses is distributed according to the GPD [Pickands (1975)]. The cdf of the GPD is given by

$$F(x \mid k, \sigma) = \begin{cases} 1 - \left(1 - \frac{kx}{\sigma}\right)^{1/k} & , k \neq 0 \\ 1 - \exp(-\frac{x}{\sigma}) & , k = 0 \end{cases}$$

where $\sigma > 0$ and $k \leq 0$, $x > 0$, while for $k > 0$ the range is $0 < x < \sigma/k$. The quantities k and σ are known as the shape and the scale parameters of the GPD, respectively. When $k < 0$ the GPD is usually known as the Pareto distribution.

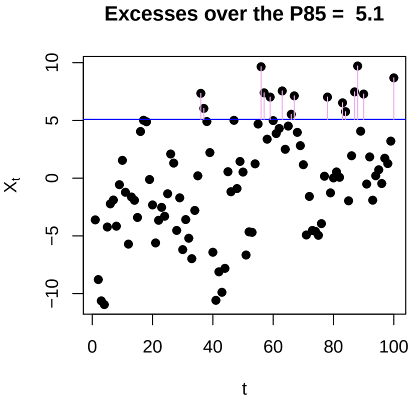

Figure 6: Simulated sample of size $n = 100$ from the model $X_i = 0.7 * X_{i-1} + e_i,\ e_i \sim N(0, \sqrt{10})$

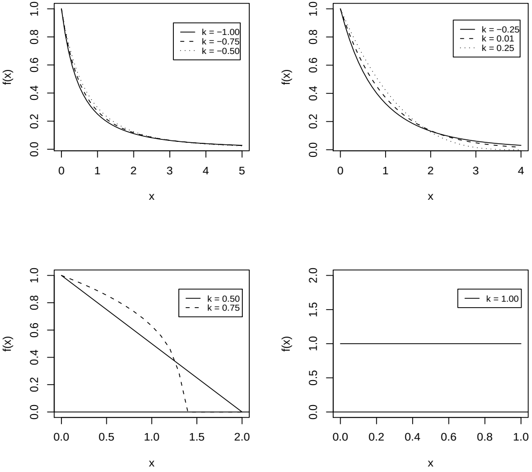

Figure 7: Pdf of the GPD for several values of k ($\sigma = 1$)

- Histogram of the data (or the histogram of the logarithm of the data if there are very large observations in the sample);
- QQ-plot;

  It consists on plotting the ascending ordered observations, $(x_{1:n}, x_{2:n}, \ldots, x_{n:n})$, *vs.* the model quantile function, $Q(p) = F^{-1}(p)$,

$$(F^{-1}(p_{i:n}), x_{i:n}), i = 1, 2, \ldots, n,$$

where $p_{i:n} = \frac{i}{n+1}$ are the plotting positions (several other choices of plotting positions are available in the literature).

1. The linearity of the QQ-plot supports the (alleged) model.
2. The non-linearity of the QQ-plot indicates that the data has a heavier or a lower tail than the model considered.

Distributions	Plot
Gumbel	$(-\log(-\log(p_{i:n})), x_{i:n})$
Exponential	$(-\log(1-p_{i:n}), x_{i:n})$
Pareto	$(-\log(1-p_{i:n}), \log(x_{i:n}))$
Normal	$(\Phi^{-1}(p_{i:n}), x_{i:n})$
Lognormal	$(\Phi^{-1}(p_{i:n}), \log(x_{i:n}))$

Table 2: Some examples

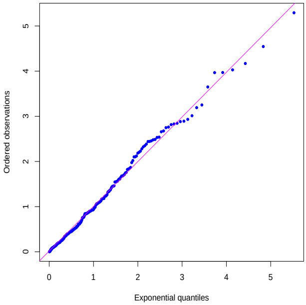

Figure 8: Exponential QQ-plot (exponential sample n = 250)

Once the parameters k and σ are estimated, the tail of the distribution can be calculated from the relation:

$$P(X > z_p) = P(X > x + u \mid X > u)P(X > u),$$

where $z_p = x + u$ is a very large number.

- the $P(X > x + u \mid X > u)$ is estimated by the GPD, as mentioned before;
- the $P(X > u)$ is, in general, estimated by $\frac{r}{n}$, where r is the number of observations in a sample of size n that exceed the threshold u.

$$z_p = \begin{cases} u + \frac{\sigma}{k}\left[1 - \left(\frac{pn}{r}\right)^k\right] & , k \neq 0 \\ u + \sigma\left(-\log \frac{pn}{r}\right) & , k = 0 \end{cases}$$

The choice of u (equivalently r) can be very tricky!

- Du Mouchel [Du Mouchel (1983)] suggests choosing 10% of the sample size as the number of upper order statistics, or, equivalently, a threshold equal to the 90*th* sample quantile;
- There are, in the literature, some algorithms to choose r (see e.g. Beirlant et al. (2004));
- Graphical
  - plot an estimator of k as a function of $u = x_{n-r:n}$ or as a function of r;
  - use the empirical mean excess function (MEF). The theoretical MEF is defined as

$$e(u) = E\left(X - u \mid X > u\right).$$

Why is the MEF used as a technique to assess the validity of the GPD model?

For the GPD(k, σ), the MEF is given by

$$e(u) = E(X - u \mid X > u) = \frac{\sigma - uk}{1 + k}, k > -1, u > 0, \sigma - ku > 0.$$

In applications, the sample MEF should be plotted as a function of

$u = x_{n-r:n}$, where r is the number of upper order statistics and is given as

$$e_n(u) = \frac{\sum_{i=1}^{n}(X_i - u)I(X_i > u)}{\sum_{i=1}^{n} I(X_i > u)},$$

where $I(.)$ is the indicator function.

(choose u such that $e_n(x)$ is approximately linear for $x \geq u$.)

- The MEF is constant for the exponential distribution;
- The MEF ultimately increases for distributions that are heavier-tailed than the exponential distribution;
- The MEF ultimately decreases for distributions that are lighter-tailed than the exponential distribution.

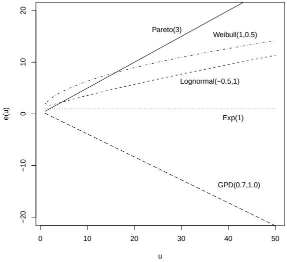

Figure 9: MEF of several common distributions (u is the threshold)

The GPD is extensively used in applications in several areas, such as:

- Engineering - turbine data, active sonar, steel industry, fatigue of materials;
- Environment - avalanches;
- Economy/Finance - interest rates, stock market indexes;
- Hydrology - river discharge, floods and droughts, sea level, wave height;
- Insurance - fire losses, windstorm losses;
- Wheather/Climatology - rainfall, dry spells, hurricanes, wind speed, wind velocity, wind speed gusts;
- Forest fires - area burned;
- Seismology - earthquake energy.

(see de Zea Bermudez and Kotz (2010 a)) for various applications).

Methods reviewed in the two papers:

- Classical methods of maximum likelihood (ML), moments (MOM) and probability weighted moments (PWM);
- Generalized versions of the PWM (GPWM) and MOM;
- *L-moments* and higher order *L-moments*;
- Partial PWM and partial *L-moments*;
- Hybrid MOM and PWM approaches;
- Penalized ML;
- The least squares;
- Robust approaches - *e.g.* the elemental percentile method (EPM) and the median estimators;
- Bayesian approaches - *e.g.* using independent Jeffreys' priors for the parameters, Pareto/Gamma for heavy-tailed distributions and Gamma/Gamma for the lighter-tailed;
- Estimation methods embedded in the POT approach.

PWM estimators

$$M_{p,r,s} = E\{X^p[F(X)]^r[1-F(X)]^s\}, r, p, s \in \mathbb{R}.$$

For the GPD(k, σ), $\sigma > 0$, it is usual to consider,

$$\alpha_s = M_{1,0,s} = E\{[X[1-F(X)]^s\} = \frac{\sigma}{(s+1)(s+1+k)},$$

where $k > -1$ and s is a non-negative (small) integer value. The following expressions are obtained:

$$k = \frac{\alpha_0}{\alpha_0 - 2\alpha_1} - 2; \sigma = \frac{2\alpha_0\alpha_1}{\alpha_0 - 2\alpha_1}.$$

The obvious estimate for α_s is $\hat{\alpha}_s = \frac{1}{n}\sum_{i=1}^{n} x_{i:n}(1-p_{i:n})^s$, for $s = 0, 1$ and the recommended plotting positions are $p_{i:n} = \frac{i-0.35}{n}, i = 1, 2, ..., n.$

GPWM estimators

Rasmussen (2001) argued that the choice indicated previously is basically driven by analytical simplicity. His proposal is to consider the GPWMs. Using the quantile function instead we get:

$$M_{1,r,s} = \int_0^1 x(F) F^r (1-F)^s dF.$$

For the GPD(k,σ) we get

$$M_{1,r,s} = \frac{\sigma}{k}\left[B(r+1, s+1) - B(r+1, s+k+1)\right],$$

where $B(.,.)$ stands for the Beta function. These moments exist provided $r > -1$, $s > -1$ and $k > -1 - s$.

GPWM estimators

$$\hat{k} = \frac{\hat{\alpha}_{s_1}(s_1+1)^2 - \hat{\alpha}_{s_2}(s_2+1)^2}{\hat{\alpha}_{s_2}(s_2+1) - \hat{\alpha}_{s_1}(s_1+1)}, s_1 > -1$$

and

$$\hat{\sigma} = \hat{\alpha}_{s_2}(s_2+1)(s_2+1+\hat{k}), s_2 > -1$$

where α_{s_1} and α_{s_2} are the PWM estimators $\alpha_s = M_{1,0,s}$.

Empirical way of choosing s_1 and s_2:

$$s_1 = \begin{cases} -0.5 & , k \leq 0 \\ k - 0.25 & , 0 < k < 0.10 \\ -0.15 & , k \geq 0.10 \end{cases}$$

$$s_2 = \begin{cases} c & , k \leq -0.25 \\ c - (k+0.25)(c+0.15)/0.35 & , -0.25 < k < 0.10 \\ -0.151 & , k \geq 0.10 \end{cases}$$

where $c = -0.92 + 0.64 \ln T$ and T is the return period.

EPM estimators - proposed by Castillo and Hadi (1997).

1. Initial estimates:

$$\hat{k}(i,j) = \frac{\ln\left(1 - \frac{x_{i:n}}{\hat{\delta}(i,j)}\right)}{C_i}$$

and

$$\hat{\sigma} = \hat{\delta}(i,j)\hat{k}(i,j)$$

where $C_i = \ln(1 - p_{i:n}) = \ln(1 - i/(n+1))$ and $\delta(i,j)$ is the solution of the equation

$$x_{i:n}\left[1 - (1 - p_{j:n})^k\right] = x_{j:n}\left[1 - (1 - p_{i:n})^k\right]$$

for every i and j $(i = 1, 2, ..., n-1)$
Authors suggestion - $j = n$.

2. Final estimates:
$\hat{k}$ = median($\hat{k}(i,n)$) and $\hat{\sigma}$ = median($\hat{\sigma}(i,n)$),i = 1, 2, ..., n-1.

Even though all the methods were never jointly compared, and we did not perform any such simulations, we tried to put together the results of separate six simulation studies (some of those quite extensive) carried out by several authors and improve the few "practical rules"that exist in the literature. Our proposed guidelines were:

- The GPWM is recommended for $k < 0$. The GPWM dominates over the PWM method for negative k (the range where PWM traditionally worked the best).

  The EPM is suggested for heavier tails ($k < -0.5$).

  The use of the GMOM (for estimating k) and the ML methods might also be an option to consider in the range,say $-0.4 \leq k < 0$. Ashkar and Tatsambon (2007) suggest ML, provided interest focuses on estimating quantiles with return period smaller than the sample size.

- LS method is the suggested method for $k > 0$ (from small to moderate sample sizes).

  Alternatively EPM might be used (for any sample size).

  According to Ashkar and Tatsambon (2007), the ML method might also be a possibility (up to $k = -0.4$ and considering that interest focuses on estimating quantiles with return period smaller than the sample size). Again, we could not definitely favor one method over the others.

- For $k > 0.2$ the ML estimation might be used.

  For larger values of k ($k > 0.4$), the EPM is possibly the best option.

Data: values of the PSI 20, at closing time, from 24 January 2000 to 29 April 2011.

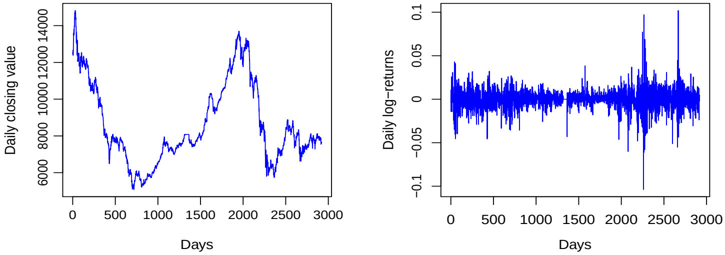

Figure 10: Daily values of the PSI 20 at closing time (left) and corresponding log-returns (right)

Log-returns: $R_t = \log\left(\frac{X_{t+1}}{X_t}\right)$, where X_t is the value of the PSI 20 at time t.

The data exhibits the usual characteristics of financial data (nonlinearity and heavy-tailed behavior).

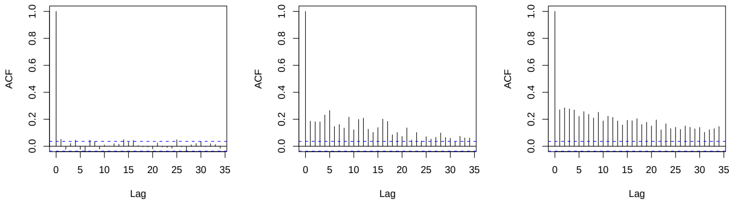

Figure 11: ACFs of the log-returns (left), of the squared log-returns (centre) and of the absolute values (right)

The log-returns were considered and the positive log-returns (right tail) were separated from the negative ones (left tail).

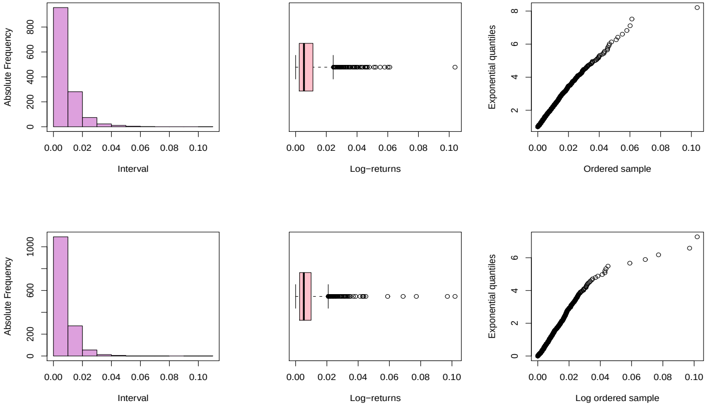

Figure 12: Histograms, Box-plots and Exponential QQ-plots of the absolute values of the negative (top row) and of the positive (bottom row) log-returns

Descriptive statistics

Statistics	Left tail	Right tail
n	1352	1445
Mean	0.008354	0.007479
St Dev	0.009036	0.007998
Min	0.000005	0.000005
$q_{0.01}$	0.000105	0.000088
$\mid x_{1:n} - q_{0.01} \mid$	0.000100	0.000083
$q_{0.99}$	0.043342	0.033341
Max	0.103792	0.101959
$x_{n:n} - q_{0.99}$	0.060450	0.068618

Table 3: Descriptive statistics - left and right tails of the log-returns

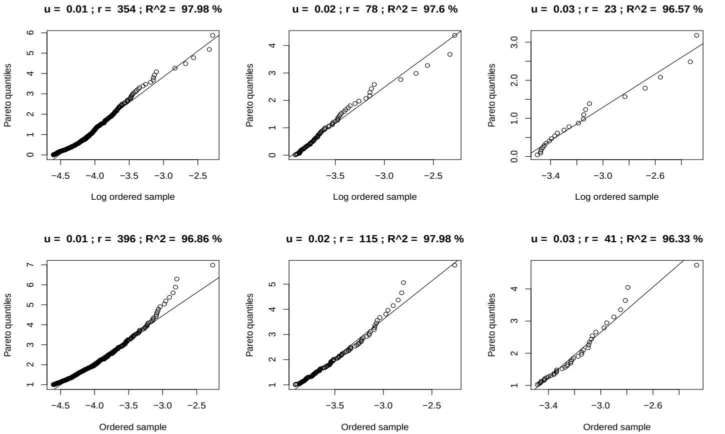

Figure 13: Pareto QQ-plots for several thresholds - positive log-returns (top row) and absolute values of the negative log-returns (bottom row)

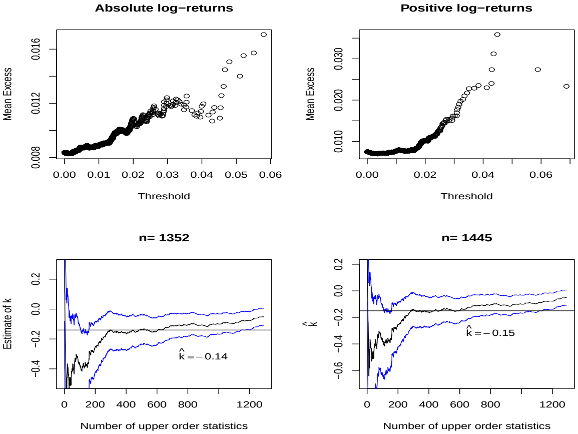

Figure 14: MEF of the absolute values (top left) and the positive (top right) log-returns; plot of the ML estimates of k the absolute values (bottom left) and the positive (bottom right) log-returns

Moments estimator for $\xi = -k$ ($k \in \mathbb{R}$) (Dekkers and de Haan (1989))

$$\hat{\xi} = M_n^{(1)} + 1 - \frac{1}{2}\left\{1 - \frac{\left[M_n^{(1)}\right]^2}{M_n^{(2)}}\right\}^{-1}$$

where $$M_n^{(2)} = \frac{1}{m}\sum_{i=0}^{m-1}(\log X_{n-i:n} - \log X_{n-m:n})^2 .$$

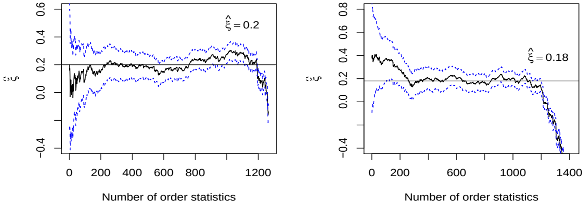

Figure 15: Plots of the estimates of $\xi = -k$ produced by the moments estimator for the absolute values (left) and for the positive (right)

Most appropriate methods: ML, (PMW) GPWM and EPM.

r	ML		EPM		PWM		GPWM	
%	$\hat{k}$	$\hat{\sigma}$	$\hat{k}$	$\hat{\sigma}$	$\hat{k}$	$\hat{\sigma}$	$\hat{k}$	$\hat{\sigma}$
200	-0.288	0.00539	-0.382	0.00508	-0.286	0.00545	-0.237	0.00589
13.8	0.083	0.00539	0.064	0.00056	0.094	0.00062	0.076	0.00071
250	-0.194	0.00620	-0.303	0.00625	-0.160	0.00654	-0.117	0.00723
17.3	0.064	0.00620	0.060	0.00050	0.075	0.00064	0.058	0.00057
275	-0.181	0.00627	-0.282	0.00654	-0.143	0.00700	-0.118	0.00716
19.0	0.060	0.00058	0.052	0.00055	0.071	0.00062	0.055	0.00059
300	-0.146	0.00672	-0.254	0.00707	-0.080	0.00700	-0.086	0.00760
20.8	0.054	0.00059	0.053	0.00052	0.066	0.00065	0.040	0.00057
325	-0.153	0.00653	-0.276	0.00641	-0.106	0.00700	-0.108	0.00721
22.5	0.053	0.00055	0.052	0.00052	0.064	0.00059	0.052	0.00052

Table 4: Models for the positive values of the log-returns of the PSI 20 (r = number of upper order statistics); r = 250 corresponds to a threshold of $u \approx 0.013$

The values in all the second lines correspond to the standard errors of the estimates. For the EPM and for the GPWM $m = 1000$ bootstrap samples were used. A return period of T = 1000 was used.

Most appropriate methods: GPWM

What are the consequences on the estimates of the GPD if other values of T are considered?

r	*Estimate*	$T = 50$	$T = 100$	$T = 1000$
200	$\hat{k}$	-0.26	-0.25	-0.24
	$\hat{\sigma}$	0.0056	0.0057	0.0059
250	$\hat{k}$	-0.18	-0.16	-0.12
	$\hat{\sigma}$	0.0065	0.0067	0.0072
275	$\hat{k}$	-0.17	-0.16	-0.12
	$\hat{\sigma}$	0.0066	0.0067	0.0072
300	$\hat{k}$	-0.15	-0.13	-0.09
	$\hat{\sigma}$	0.0069	0.0071	0.0076
325	$\hat{k}$	-0.15	-0.14	-0.11
	$\hat{\sigma}$	0.0067	0.00688	0.0072

Table 5: Models for the positive values of the log-returns of the PSI 20 considering other return periods

Most appropriate methods: ML, (PWM) GPWM and EPM.

	ML		EPM		PWM		GPWM	
r	$\hat{k}$	$\hat{\sigma}$	$\hat{k}$	$\hat{\sigma}$	$\hat{k}$	$\hat{\sigma}$	$\hat{k}$	$\hat{\sigma}$
50	-0.094	0.01015	-0.342	0.00904	-0.106	0.01003	-0.109	0.00977
3.7	0.149	0.01015	0.010	0.00277	0.164	0.00218	0.011	0.00304
75	-0.086	0.01001	-0.298	0.00897	-0.098	0.00988	-0.095	0.00977
5.5	0.122	0.01001	0.010	0.00201	0.133	0.00176	0.010	0.00220
100	-0.107	0.00937	-0.289	0.00848	-0.126	0.00917	-0.125	0.00898
7.4	0.112	0.00937	0.009	0.00166	0.116	0.00142	0.009	0.00179
125	-0.079	0.00968	-0.256	0.00881	-0.081	0.00967	-0.073	0.00972
9.2	0.093	0.00968	0.010	0.00147	0.103	0.00133	0.010	0.00145
150	-0.142	0.00843	-0.280	0.00780	-0.165	0.00819	-0.173	0.00784
11.1	0.099	0.00843	0.008	0.00122	0.097	0.00104	0.008	0.00132
200	-0.085	0.00910	-0.236	0.00838	-0.080	0.00915	-0.072	0.00925
14.8	0.074	0.00910	0.009	0.00107	0.081	0.00099	0.009	0.00094

Table 6: Models for the absolute values of the negative log-returns of the PSI 20 (r = number of upper order statistics; r = 50 corresponds to a threshold of $u \approx 0.029$

The values in all the second lines correspond to the standard errors of the estimates. For the EPM and for the GPWM $m = 1000$ bootstrap samples were used. A return period of T = 1000 was used.

Most appropriate methods: GPWM

What are the consequences on the estimates of the GPD if other values of T are considered?

r	*Estimate*	$T = 50$	$T = 100$	$T = 1000$
50	$\hat{k}$	-0.083	-0.087	-0.109
	$\hat{\sigma}$	0.010	0.010	0.011
75	$\hat{k}$	-0.080	-0.082	-0.095
	$\hat{\sigma}$	0.010	0.010	0.010
100	$\hat{k}$	-0.105	-0.110	-0.125
	$\hat{\sigma}$	0.009	0.009	0.009
125	$\hat{k}$	-0.074	-0.074	-0.073
	$\hat{\sigma}$	0.010	0.010	0.010
150	$\hat{k}$	-0.143	-0.151	-0.173
	$\hat{\sigma}$	0.008	0.008	0.008
200	$\hat{k}$	-0.080	-0.079	-0.072
	$\hat{\sigma}$	0.009	0.009	0.009

Table 7: Models for the negative log-returns of the PSI 20 considering other return periods

GPWM extreme quantile estimation:

	Tail probability					
Left tail	0.01	0.005	0.004	0.003	0.002	0.001
Log-return	0.0425	0.0507	0.0535	0.0572	0.0627	0.0726
Empirical log-return	0.0433	0.0487	0.0517	0.0549	0.0586	0.0609
Right tail	0.01	0.005	0.004	0.003	0.002	0.001
Log-return	0.0371	0.0444	0.0468	0.0501	0.0550	0.0638
Empirical log-return	0.0333	0.0430	0.0439	0.0542	0.0697	0.0883

Table 8: Extreme quantile estimation

Final comments:

- The GPD is a very important distribution in the EVT framework as an alternative to the classical methods, namely because it uses the data more efficiently;
- The problem lies in an proper choice of the threshold. There has to be a "trade-off"between bias and variance of the parameter estimators;
- This application shows that the proposed "guidelines"seem to be accurate, although no joint simulation was carried out $\rightarrow$ Future work.

Thank you very much for your attention :)))